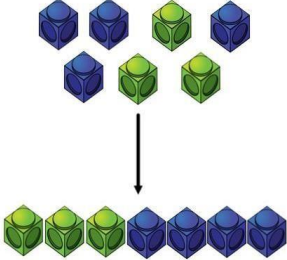
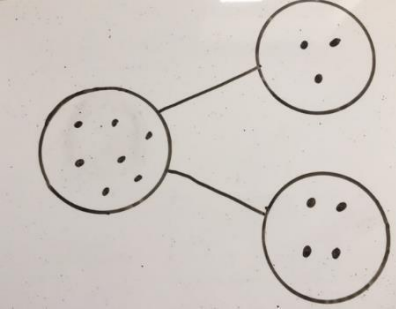
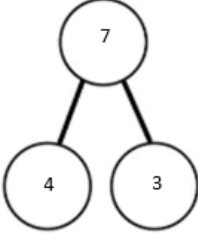
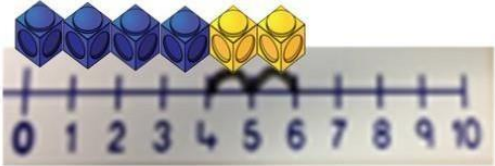
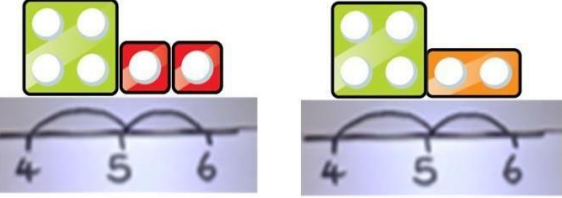
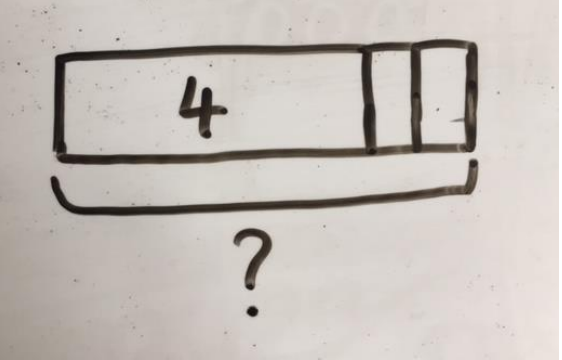
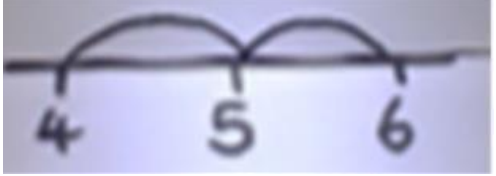




Key language: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to' 'is the same as'.

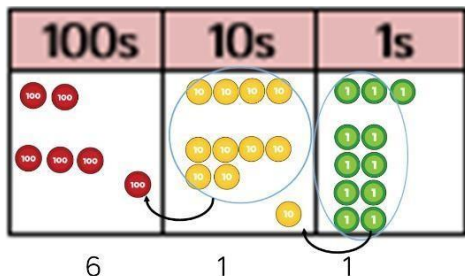
Concrete	Pictorial	Abstract
<p>Combining two parts to make a whole (use other resources too e.g. eggs, shells, teddy bears, cars).</p> 	<p>Children to represent the cubes using dots or crosses. They could put each part on a part whole model too.</p> 	<p>$4 + 3 = 7$ Four is a part, 3 is a part and the whole is seven.</p> 
<p>Counting on using number lines using cubes or images</p>  	<p>A bar model which encourages the children to count on, rather than count all.</p> 	<p>The abstract number line: What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? $4 + 2$</p> 



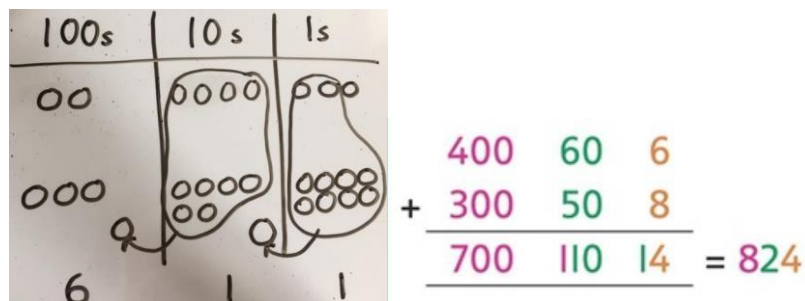
<p>Regrouping to make 10; using ten frames and counters/cubes. 6 + 5</p>	<p>Children to draw the ten frame and counters/cubes.</p>	<p>Children to develop an understanding of equality e.g.</p> $6 + \square = 11$ $6 + 5 = 5 + \square$ $6 + 5 = \square + 4$
<p>TO + O using base 10. Continue to develop understanding of partitioning and place value. 41 + 8</p>	<p>Children to represent the base 10 e.g. lines for tens and dot/crosses for ones.</p>	<p>41 + 8</p> <p>1 + 8 = 9 40 + 9 = 49</p>
<p>TO + TO using base 10. Continue to develop understanding of partitioning and place value. 36 + 25</p>	<p>Children to represent the base 10 in a place value chart.</p>	<p>Looking for ways to make 10.</p> <p>36 + 25 =</p> <p>30 + 20 = 50 5 + 5 = 10 50 + 10 + 1 = 61</p> <p>1 5 36</p> <p>Formal method:</p> $\begin{array}{r} +25 \\ 36 \\ \hline 61 \\ \hline 1 \end{array}$



Use of place value counters to add HTO+ TO, HTO+ HTO etc. When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred.

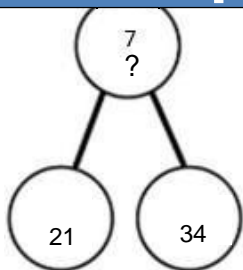


Children to represent the counters in a place value chart, circling when they make an exchange. Moving to expanded written method.



$$\begin{array}{r}
 243 \\
 +368 \\
 \hline
 611 \\
 \hline
 1 \quad 1
 \end{array}$$

Conceptual variation; different ways to ask children to solve 21 + 34



?	
21	34

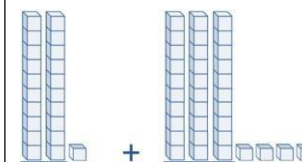
Word problems:
In year 3, there are 21 children and in year 4, there are 34 children.
How many children in total?

$21 + 34 = 55$. Prove it

$$\begin{array}{r}
 21 \\
 +34 \\
 \hline
 \end{array}$$

$21 + 34 =$
 = 21 + 34

Calculate the sum of twenty-one and thirty-four.

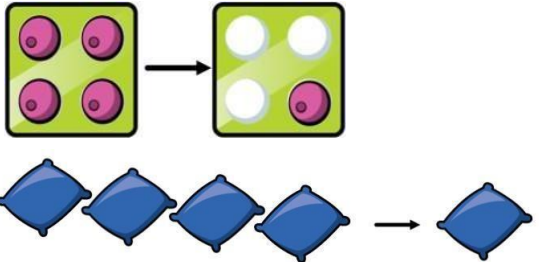
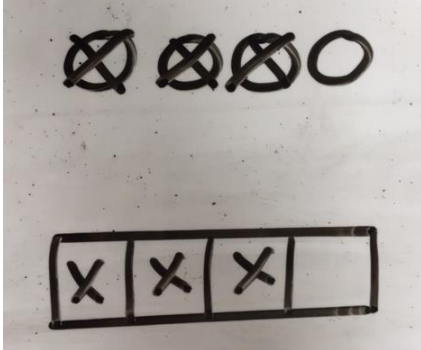

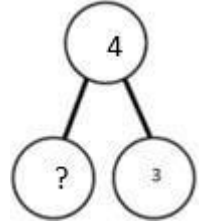
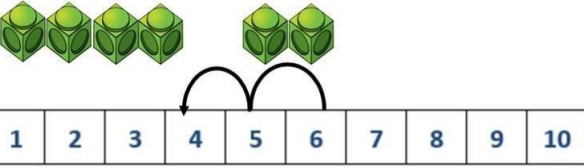
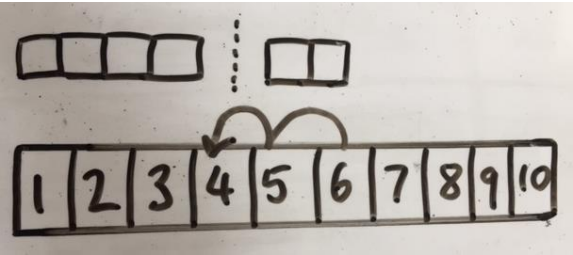
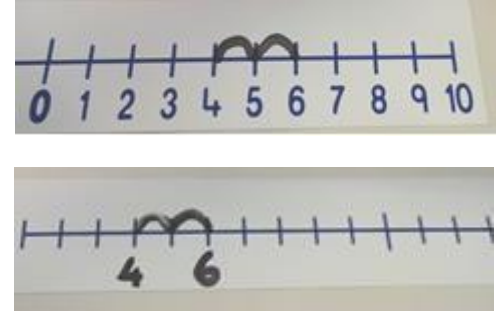


Missing digit problems:

10s	1s
10 10	1
10 10 10	?
?	5



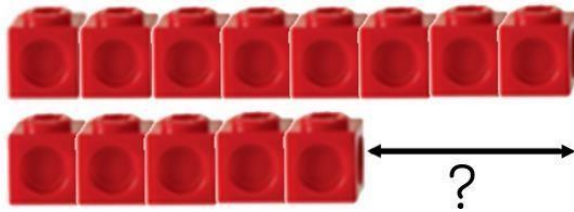
Key language: take away, less than, the difference, subtract, minus, fewer, decrease.

Concrete	Pictorial	Abstract				
<p>Physically taking away and removing objects from a whole (tenframes, Numicon, cubes and other items such as beanbags could be used).</p> <p>$4 - 3 = 1$</p> 	<p>Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used.</p> 	<p>$4 - 3 =$</p> <p> $= 4 - 3$</p> <table border="1" data-bbox="1646 550 1960 630"> <tr> <td colspan="2">4</td> </tr> <tr> <td>3</td> <td>?</td> </tr> </table> 	4		3	?
4						
3	?					
<p>Counting back (using numberlines or numbertracks) children start with 6 and count back 2.</p> <p>$6 - 2 = 4$</p> 	<p>Children to represent what they see pictorially e.g.</p> 	<p>Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line</p> 				

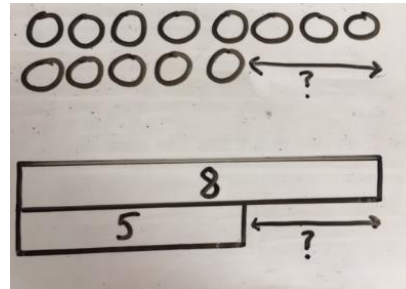


Finding the difference (using cubes, Numicon or Cuisenaire rods, other objects can also be used).

Calculate the difference between 8 and 5.



Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.



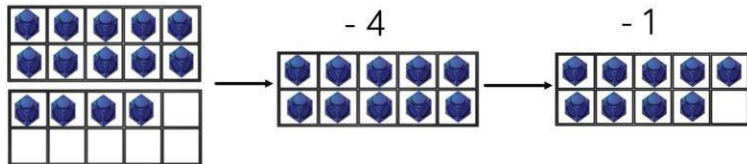
Find the difference between 8 and 5.

$8 - 5$, the difference

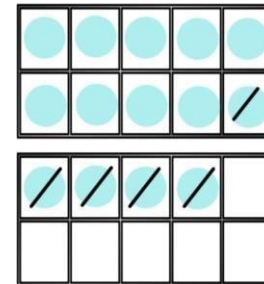
Children to explore why $9 - 6 = 8 - 5 = 7 - 4$ have the same difference.

Making 10 using ten frames.

$14 - 5$



Children to present the ten frame pictorially and discuss what they did to make 10.



Children to show how they can make 10 by partitioning the subtrahend.

$$14 - 5 = 9$$

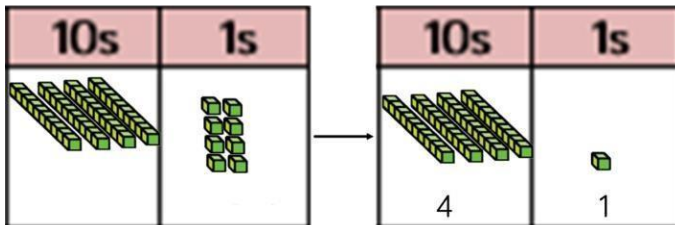
4 1

$$14 - 4 = 10$$

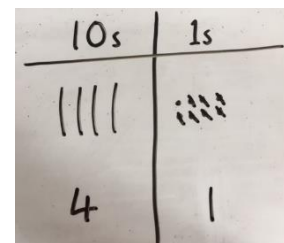
$$10 - 1 = 9$$

Column method using base 10.

$48 - 7$



Children to represent the base 10 pictorially.

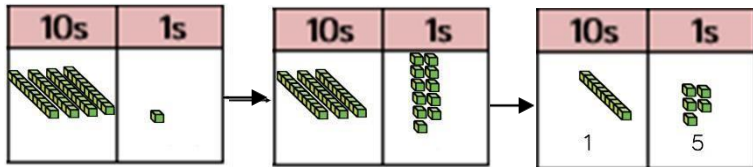


Column method or children could count back 7.

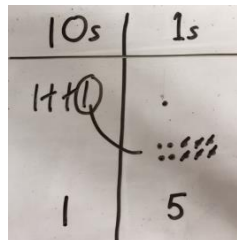
	4	8
-		7
	4	1



Column method using base 10 and having to exchange.
41 – 26



Represent the base 10 pictorially, remembering to show the exchange.

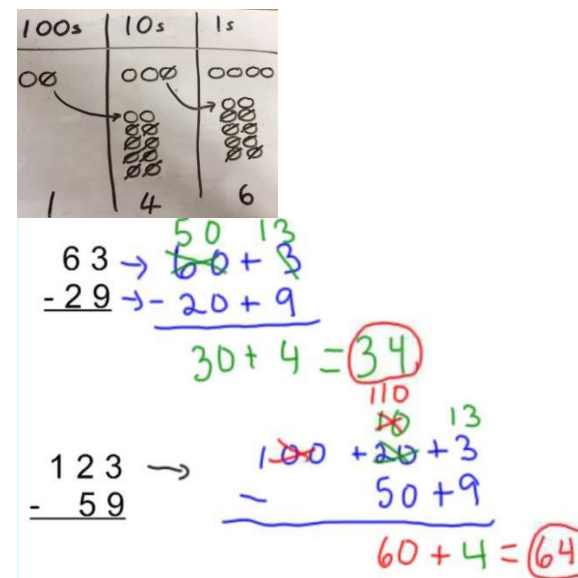


Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because $41 = 30 + 11$.

$$\begin{array}{r} 3 \cancel{4} 1 \\ - 26 \\ \hline 15 \end{array}$$

Column method using placevalue counters.
234 – 88

Represent the placevalue counters pictorially; remembering to show what has been exchanged, moving to expanded method.



Formal column method. Children must understand what has happened when they have crossed out digits.

$$\begin{array}{r} 2 \quad 1 \\ \cancel{2} \cancel{3} 4 \\ - 88 \\ \hline 6 \end{array}$$

Conceptual variation; different ways to ask children to solve 391 – 186



391	
?	186

391	
186	?

Raj spent £391, Timmy spent £186.
How much more did Raj spend?

Calculate the difference between 391 and 186.

= 391 – 186

$$\begin{array}{r} 391 \\ -186 \\ \hline \end{array}$$

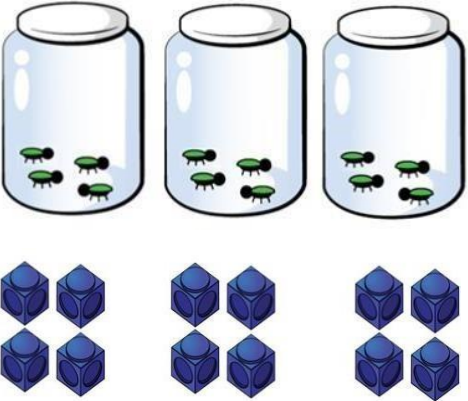
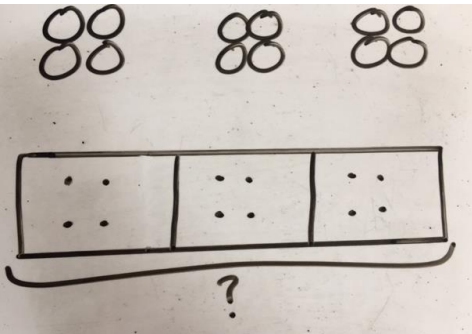
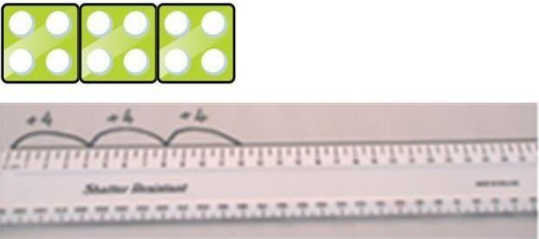
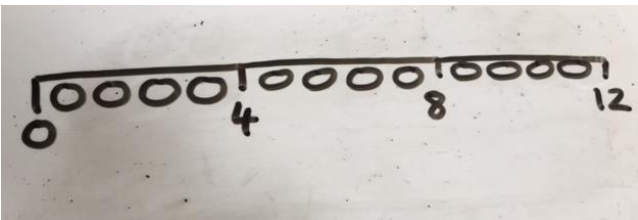
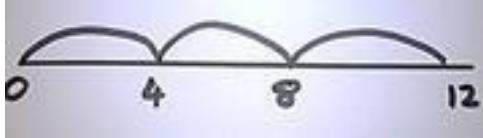
What is 186 less than 391?

Missing digit calculations

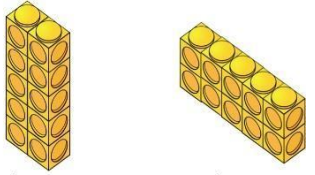
$$\begin{array}{r} 39\Box \\ -\Box\Box6 \\ \hline \Box05 \end{array}$$



Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups.

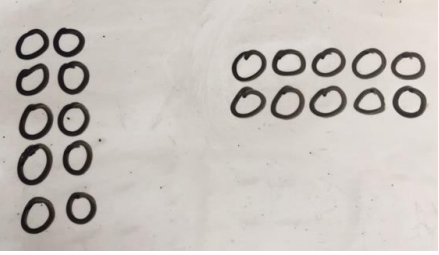
Concrete	Pictorial	Abstract
<p>Repeated grouping/repeated addition 3×4 $4 + 4 + 4$ There are 3 equal groups, with 4 in each group.</p> 	<p>Children to represent the practical resources in a picture and use a bar model.</p> 	<p>$3 \times 4 = 12$ $4 + 4 + 4 = 12$</p>
<p>Number lines to show repeated groups- 3×4</p>  <p>Cuisenaire rods can be used too.</p>	<p>Represent this pictorially alongside a number line e.g.:</p> 	<p>Abstract number line showing three jumps of four.</p> <p>$3 \times 4 = 12$</p> 

Use arrays to illustrate commutativity counters and other objects can also be used.
 $2 \times 5 = 5 \times 2$



2 lots of 5 5 lots of 2

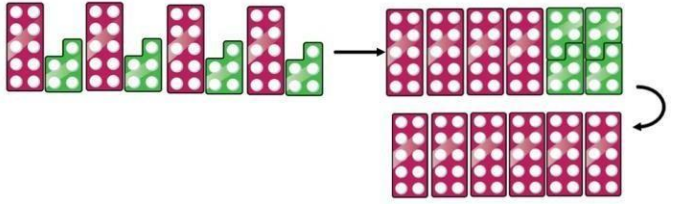
Children to represent the arrays pictorially.



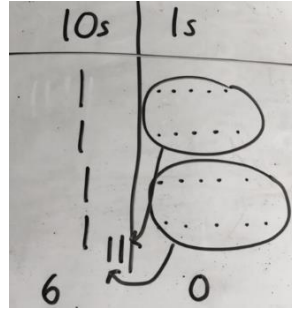
Children to be able to use an array to write a range of calculations e.g.

$10 = 2 \times 5$
 $5 \times 2 = 10$
 $2 + 2 + 2 + 2 + 2 = 10$
 $10 = 5 + 5$

Partition to multiply base 10 or Cuisenaire rods.
 4×15



Children to represent the concrete manipulatives pictorially.

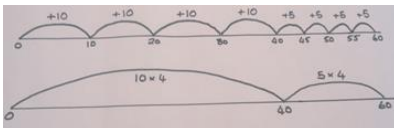


Children to be encouraged to show the steps they have taken.



4×15
 $\swarrow \searrow$
 $10 \quad 5$

$10 \times 4 = 40$
 $5 \times 4 = 20$
 $40 + 20 = 60$

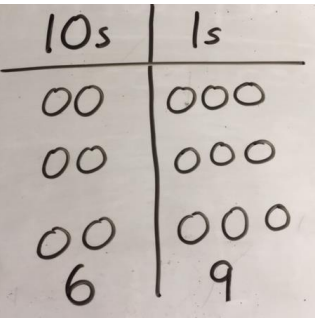
A number line can also be used



Formal column method with place value counters (base 10 can also be used.) 3×23

10s	1s
	
6	9

Children to represent the counters pictorially.



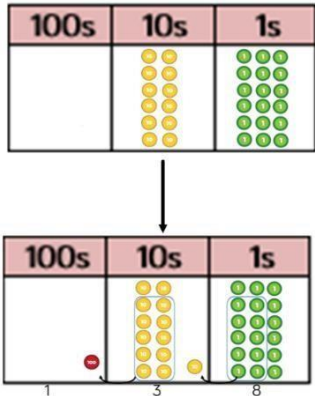
Children to record what it is they are doing to show understanding.

$\begin{array}{r} 54 \\ \times 6 \\ \hline 24 \\ + 300 \\ \hline 324 \end{array}$	$\begin{array}{r} 23 \\ \times 3 \\ \hline 69 \end{array}$
-----------------------------------------------------------------------------------	------------------------------------------------------------

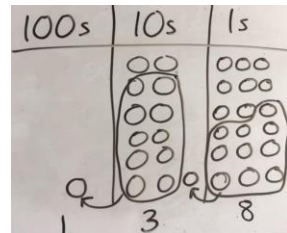
3×23 $3 \times 20 = 60$
 $3 \times 3 = 9$
 $60 + 9 = 69$



Formal column method with place value counters. 6×23

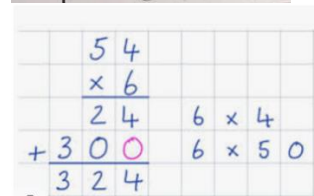


Children to represent the counters/base 10, pictorially e.g. the image below.



Children to record what it is they are doing to show understanding.

$$3 \times 23 = 3 \times 20 + 3 \times 3 = 60 + 9 = 69$$



Formal written method

$$\begin{array}{r} 6 \times 23 = \\ 23 \\ \times 6 \\ \hline 138 \end{array}$$

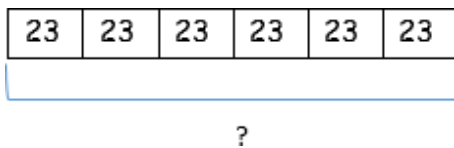
When children start to multiply $3d \times 3d$ and $4d \times 2d$ etc., they should be confident with the abstract:

To get 744 children have solved 6×124 .
To get 2480 they have solved 20×124 .

$$\begin{array}{r} 124 \\ \times 26 \\ \hline 744 \\ 2480 \\ \hline 3224 \end{array}$$

Answer: 3224

Conceptual variation; different ways to ask children to solve 6×23



Mai had to swim 23 lengths, 6 times a week.
How many lengths did she swim in one week?

With the counters, prove that $6 \times 23 = 138$

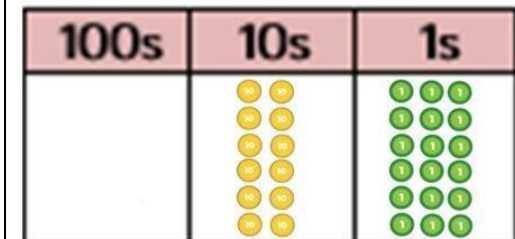
Find the product of 6 and 23

$$6 \times 23 =$$

$$\square = 6 \times 23$$

$$\begin{array}{r} 6 \quad 23 \\ \times 23 \quad \times 6 \\ \hline \quad \quad \end{array}$$

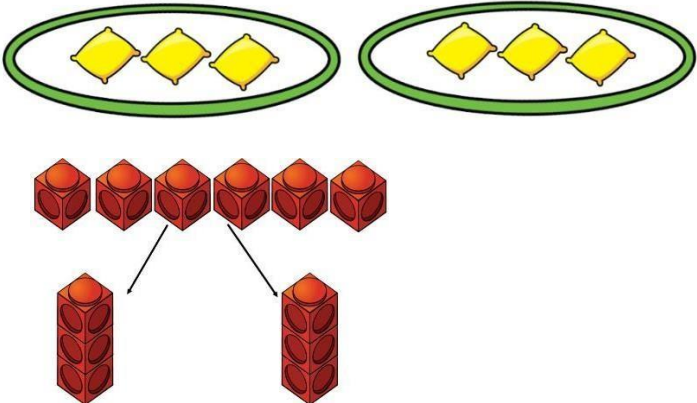
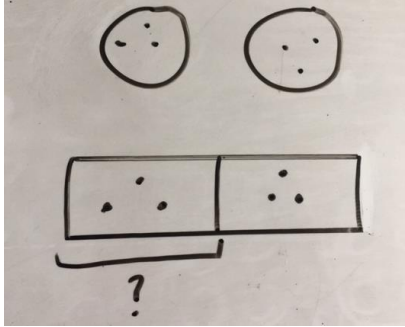
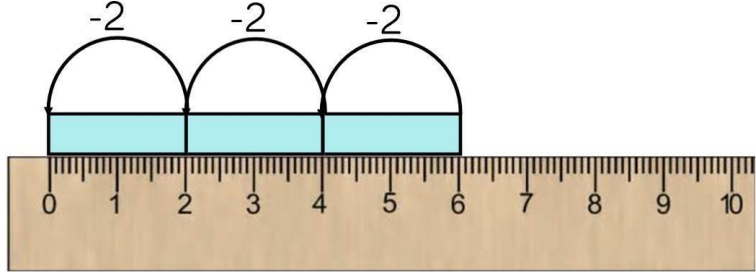
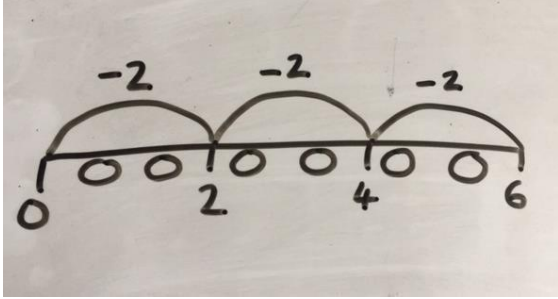
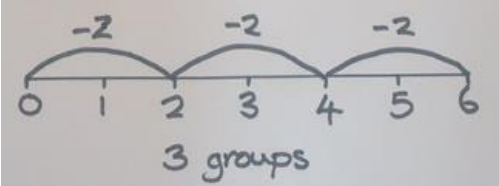
What is the calculation?
What is the product?





Calculation policy: Division

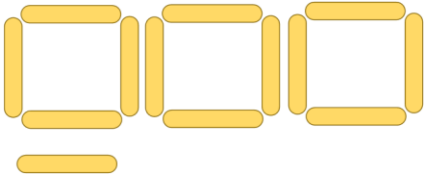
Key language: share, group, divide, divided by, half.

Concrete	Pictorial	Abstract		
<p>Sharing using a range of objects. $6 \div 2$</p> 	<p>Represent the sharing pictorially.</p> 	<p>$6 \div 2 = 3$</p> <table border="1" data-bbox="1554 459 2007 528"> <tr> <td>3</td> <td>3</td> </tr> </table> <p>Children should also be encouraged to use their 2 times tables facts.</p>	3	3
3	3			
<p>Repeated subtraction using Cuisenaire rods above a ruler. $6 \div 2$</p>  <p>3 groups of 2</p>	<p>Children to represent repeated subtraction pictorially.</p> 	<p>Abstract number line to represent the equal groups that have been subtracted.</p> 		



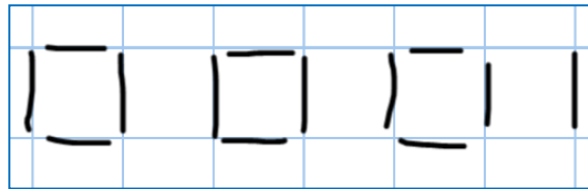
2d ÷ 1d with remainders using lollipop sticks. Cuisenaire rods, above a ruler can also be used.
 $13 \div 4$

Use of lollipop sticks to form wholes - squares are made because we are dividing by 4.



There are 3 whole squares, with 1 left over.

Children to represent the lollipop sticks pictorially.

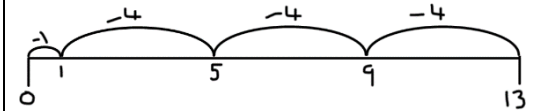


There are 3 whole squares, with 1 left over.

$13 \div 4 = 3$ remainder 1

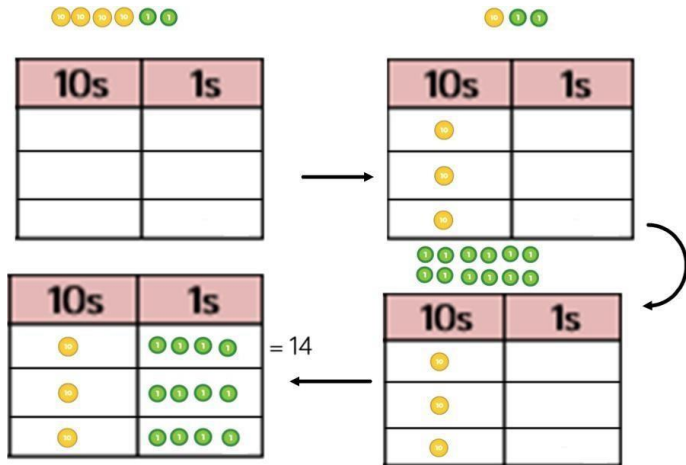
Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line.

'3 groups of 4, with 1 left over'

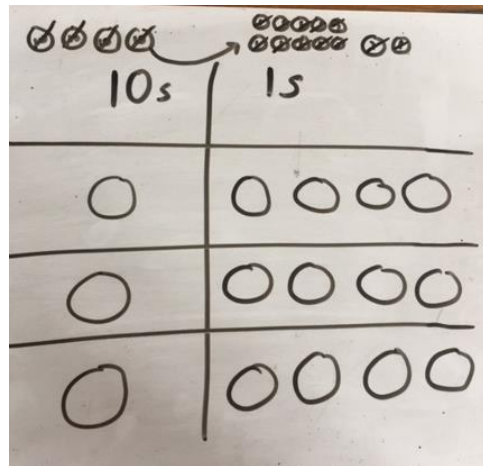


Sharing using place value counters.

$42 \div 3 = 14$



Children to represent the place value counters



Children to be able to make sense of the place value counters and write calculations to show the process.

$42 \div 3$
 $42 = 30 + 12$
 $30 \div 3 = 10$
 $12 \div 3 = 4$
 $10 + 4 = 14$

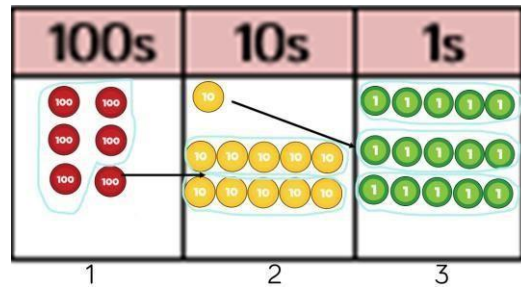
155
- 50 (10 x 5)
105
- 50 (10 x 5)
55
- 50 (10 x 5)
5
- 5 (1 x 5)
0
Therefore 155 ÷ 5 = 31

31 groups of 5 have been subtracted



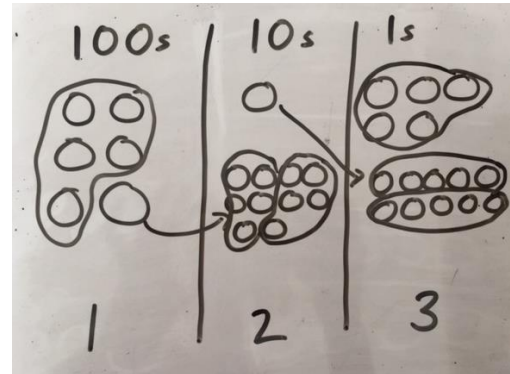
Short division using place value counterstogroup.

$$615 \div 5$$



1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

Represent the place value counters pictorially.

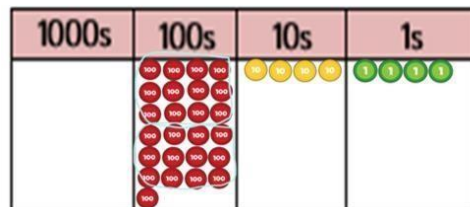


Children to the calculation using the short division scaffold.

$$\begin{array}{r} 123 \\ 5 \overline{) 615} \\ \underline{5} \\ 11 \\ \underline{10} \\ 15 \\ \underline{15} \\ 0 \end{array}$$

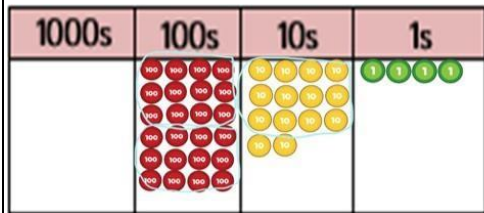
Long division using place value counters

$$2544 \div 12$$



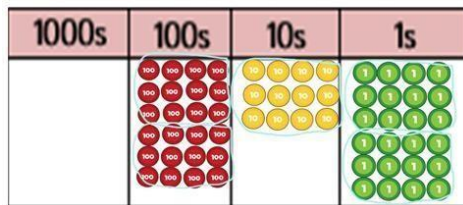
We can group 24 hundreds into groups of 12 which leaves with 1 hundred.

$$\begin{array}{r} 02 \\ 12 \overline{) 2544} \\ \underline{24} \\ 1 \end{array}$$



After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

$$\begin{array}{r} 021 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 2 \end{array}$$

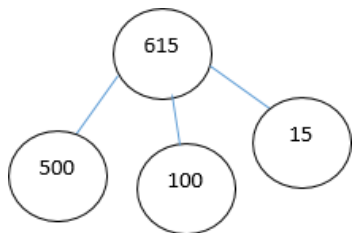


After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 group of 12, which leaves no remainder.

$$\begin{array}{r} 0212 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

Conceptual variation; different ways to ask children to solve $615 \div 5$

Using the part whole model below, how can you divide 615 by 5 without using short division?



I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

$$5 \overline{) 615}$$

$$615 \div 5 =$$

$$\square = 615 \div 5$$

What is the calculation?
What is the answer?

